Preliminary analysis of ship manoeuvrability criteria in wind

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ABSTRACT: The problem of standardizing ship manoeuvring qualities under wind action is discussed in detail. The most general formulation is analysed and further reduced to the problem of equilibrium in steady motion for which an analytic solution was obtained in the case of a linearized ship mathematical model. Unlike in other similar solutions, relationships between the parameters of the relative wind and the ship course/heading and the absolute wind speed were explicitly established. Numerical results obtained for a generic mathematical model illustrate qualitative dependence of the degree of ship controllability in wind on its degree of directional stability and configuration of the windage area.

1 INTRODUCTION
There is no need to prove importance of good manoeuvring qualities of surface displacement ships for navigational safety, operational effectiveness and—to a lesser extent—for energy efficiency. The former are evident immediately and the latter becomes so after one recalls that, for instance, a directionally unstable ship may be prone to excessive yawing in straight course which would result in a longer path and resistance increased from augmented average drift and rudder angles.

As recently much attention is paid to efficiency and safety of sea-going ships in adverse weather conditions, a necessity to revisit issues of ship behaviour in heavy weather and, possibly, work out some improved and more stringent manoeuvrability criteria. The objective of the present study was to perform primary analysis of the problem with emphasis to controllability in wind and to reveal certain qualitative peculiarities using simplified mathematical models which will facilitate more detailed and accurate analysis in the future.

Attempts to work out well defined and quantifiable manoeuvring criteria and to base on them certain manoeuvring standards have been undertaken many times since, approximately, beginning of 60s (Gertler & Gover 1961). The overall number of publications in this area is considerable, see e.g. (Yudin 1967, Tumashik 1976, Nawrocki 1977, Sobolev 1978, Amerongen & Price 1979, Baquero 1982) as characteristic examples and see also a more detailed review in (Sutulo 1995b). Worth mentioning are also studies carried out for the US Coast Guard by Barr et al. (1981) and proposals by Vassalos & Spyrou (1991). However, according to the authors’ best knowledge in only 3 cases such studies resulted in some official requirements.

First, these were the “Interim standards for effectiveness of rudders and steering nozzles” introduced in 1979 by the Russian Maritime Register of Shipping and which since 1986 and up to now constitute part of the corresponding Rules (RS 2014). Although these effectiveness requirements were formulated in such a way that they do not show explicitly manoeuvrability criteria, some of these had been in fact laid in their foundation by Mastushkin (1981), the principle developer of those requirements.

The second case are IMO manoeuvring standards (IMO 2002) first implemented in their interim version in 1993 are using explicit manoeuvring criteria and are more advanced in many respects.

Finally, recently the STANAG manoeuvring standards have been developed for NATO naval ships (Quadvlieg et al. 2010).

Even a superficial analysis of all proposed manoeuvring criteria, were they implemented or not, makes clear that they can be divided into two different groups:

1. Inherent or proper criteria independent of external disturbances and quantifying the inherent directional (in)stability of the ship, its turning ability, ability of a fast course change and yaw checking capability.

2. Environment-dependent criteria describing capability of a vessel to remain under control and counteract external disturbances, such as current, wind and hydrodynamic interaction with bodies in close proximity.

While criteria of the first group can be traced in all sets of standards, two “environmental” criteria are only present, though implicitly, in the Rules of the Russian Register (RS 2014). Both criteria account for the wind action and one of them—also for the
hydodynamic interaction. More detailed, though still concise, description of these criteria can be found in (Sutulo 1996). In other cases, it was probably assumed that a ship with sufficient inherent manoeuvrability would behave adequately and conserve controllability in typical environmental conditions.

Recently, in the framework of a coordinated research aimed at augmenting safety of ships operating in adverse weather conditions it has become clear that environmental manoeuvrability criteria are very important for the overall performance of a ship and deserve detailed consideration. It must be emphasized that such criteria are now interpreted in a more general sense also including requirements to the ship powering which must be sufficient for withstanding adverse condition.

Importance of such studies is also supported by evident incompleteness of any set of manoeuvring criteria based exclusively on the inherent ship dynamic properties like the IMO criteria which are in general reasonable in all other respects.

The following exogenous factors acting on ship in normal operation can be considered:

1. Influence of the Coriolis force originating from the rotation of the Earth.
2. Wind action.
3. Excitation from sea waves.
4. Influence of the current.
5. Hydrodynamic interaction.

The Coriolis force effects can be neglected being only significant for objects moving at very slow speed i.e. when the values of the Rossby number are small. Practically, it can be the case for freely floating objects including ships deprived of propulsion.

The remaining four factors are all significant but only the wind action will be considered here in more detail although possibilities to approximately include influence of the current and waves will be briefly discussed.

2 MATHEMATICAL MODELS AND FORMULATIONS

2.1 Frames of reference and kinematic parameters

Let \( O \zeta \eta \zeta \) be the Earth-fixed Cartesian frame with the \( \zeta \)-axis pointing downwards and let \( C \chi \psi \zeta \) be the “manoeuvring” frame fixed to the moving vessel but not involved into the motions of heave, pitch and roll (Fig. 1). The heave and pitch motions can be neglected in many cases and then at any time moment \( t \) the instantaneous position of the ship will be by the linear coordinates \( \zeta(t), \psi(t) \) and by the heading angle \( \psi(t) \) and roll angle \( \phi(t) \). The motion will be described by \( u, v, p, r \) being the quasi-velocities of surge, sway, roll and yaw respectively. All of them can take subscripts depending on which ship velocity vector they are related to. Considered are the following ship velocities: \( \mathbf{V} \)—with respect to water, \( \mathbf{V}_A \)—relative to air, \( \mathbf{V}_C \)—over the ground.

Additional kinematic parameters shown in Figure 1 are: the drift angle \( \beta \), the course angle \( \chi = \psi - \beta \), the air drift angle \( \beta_A \), the absolute wind velocity \( \mathbf{V}_W \) with the wind angle \( \chi_W \), and, finally the absolute current velocity \( \mathbf{V}_C \) with the current angle \( \chi_C \).

2.2 Equations of motion

The kinematic equations for a 4DOF model and introduced frames and parameters are:

\[
\begin{align*}
\dot{\zeta} &= u_G \cos \psi - v_G \sin \psi, \\
\dot{\psi} &= u_G \sin \psi + v_G \cos \psi, \\
\dot{\psi} &= r, \\
\dot{\phi} &= p,
\end{align*}
\]

with the auxiliary relations:

\[
\mathbf{V}_G = \mathbf{V} + \mathbf{V}_C; \quad \mathbf{V}_A = \mathbf{V}_G - \mathbf{V}_W.
\]

The corresponding equations of ship dynamics are:

\[
(m + \mu_1) \ddot{u} - mv_r - m x_G \dot{r} = X_g(\phi, u, v, r, \delta_R) + X_p(u, n)
\]

\[
+ X_A(u, v, r, \phi, V_A, \beta_A),
\]

\[
(m + \mu_2) \ddot{v} + (m x_G + \mu_2) \dot{r} + mv_r = Y_g(\phi, u, v, r, \delta_R) + Y_A(u, v, r, \phi, V_A, \beta_A),
\]

\[
(I_{zz} + \mu_6) \ddot{r} + (m x_G + \mu_6) \dot{r} + m x_G \omega_r = N_g(\phi, u, v, r, \delta_R) + N_A(u, v, r, \phi, V_A, \beta_A),
\]

\[
(I_{xx} + \mu_4) \ddot{p} + (m z_G + \mu_4) \dot{p} = K_g(\phi, u, v, r, \delta_R) + K_A(u, v, r, \phi, V_A, \beta_A).
\]

Figure 1. Frames of reference and velocities in wind and current.
where \( m \) is the ship mass, \( x_g, z_g \) are the coordinates of its centre in the moving frame assumed to be constant, \( I_{xx}, I_{zz} \) are the moments of inertia, \( \rho_i \) are the added mass coefficients, \( \delta_R \) is the rudder deflection angle; \( n \) is the propeller rotation frequency; \( X, Y, N, K \) stand for forces/moments of surge, sway, yaw and roll respectively with the subscripts meaning: \( q \)–quasi-steady hydrodynamic, \( P \)–propeller, \( A \)–aerodynamic.

The equations (1) through (3) describe arbitrary motion of a ship in wind at any given control programme \( \delta_R(t) \). Practically the same equations are valid for tall ships and sailing boats if \( X_P \equiv 0 \) and, besides the rudder angle, additional control parameters defining the sails configuration are added. These parameters can be numerous and a large dimension of the control vector makes analysis of dynamics of tall ships extremely complex.

As to the engine-powered vessels, analysis of their controllability is relatively simple and, in the case of more or less complete mathematical models is typically carried out through simulation of certain testing/standard manoeuvres. While in absence of wind any such manoeuvre can be executed by an inherently controllable ship, this is no longer true when the wind is sufficiently strong. For instance, it is well known that, say, a \( 5^\circ \)–\( 5^\circ \) zigzag can be non-executable even under moderate wind as larger helms are required for course changes.

This rather obvious fact inspired proposals of criteria for controllability in wind based on the ability or inability of a ship to perform and complete some pre-chosen test manoeuvre. For instance, Sobolev (1977) suggested to classify the ship as controllable at a given wind speed if it is able to complete a full circle from any initial heading at maximum helm. Of course a true circle is not possible in wind and it is meant here that the heading of the ship can be continuously altered by \( 360^\circ \). The maximum wind speed at which the ship conserves its controllability can serve as basis for working out the corresponding criterion and standard. Such a criterion would be reasonable from the viewpoint of practical ship handling but is somewhat difficult to verify as all initial headings must be tested.

However, Pershitz (1983) demonstrated that the capability of a ship to maintain any straight course at a given wind implies its ability to make a turn while the opposite may be not true. This becomes clear from the obvious fact that a sequence of straight courses is equivalent to turning with an infinitely low turning rate. A normal turn with a finite rate of yaw is easier to perform as a ship can gain inertia helping it to pass through unfavourable headings. The straight run criterion looks simpler and more observable and thus should be given preference.

In the straight run under steady wind \( \dot{u} = \dot{v} = \dot{r} = \dot{\rho} = \rho = 0 \) and the ship dynamics equations reduce to:

\[
X_q(\varphi,u,v,\delta_R) + X_P(u,n) + X_A(u,v,\varphi,V_A,\beta_A) = 0,
\]
\[
Y_q(\varphi,u,v,\delta_R) + Y_A(u,v,\varphi,V_A,\beta_A) = 0,
\]
\[
N_q(\varphi,u,v,\delta_R) + N_A(u,v,\varphi,V_A,\beta_A) = 0,
\]
\[
K_q(\varphi,u,v,\delta_R) + K_A(u,v,\varphi,V_A,\beta_A) = 0
\]

and the kinematic equations (1) can be effectively dropped as the first two of them are uncoupled from dynamic equations and the remaining two just show that the heading and roll angles remain constant.

The set (4) completed with additional control parameters can be used for analysis of the straight motion of sail ships for which typically the steady roll (heel) cannot be neglected. In the case of engine-powered ships the influence of steady heel is relatively small, hence, the roll equilibrium equation be dropped and it can be assumed \( \varphi = 0 \) in the remaining three equilibrium equations.

Under these simplifications, if the heading angle is pre-defined, i.e. \( \varphi = \varphi_0 \), the set (4) gives for every value of the wind speed \( V_w \) gives three nonlinear algebraic equations for three equilibrium parameters: \( u_0, v_0, \delta_R0 \) at which the ship will be balanced for the required straight run. If for some \( \varphi_0 \) the solution does not exist, it will mean that the ship will not be able to maintain straight course and thus the controllability will be lost.

In general, equations (4) are not very easy to solve and the algorithm must be specific for any particular mathematical model. For instance, such equations were solved in (Sutulo 1985) by means of a specially designed iterative procedure. The problem can, however, be significantly simplified in the particular case of linearized equilibrium equations.

2.3 Linear analysis of steady motion in wind

It is well known that manoeuvring mathematical models of surface displacement ships are substantially nonlinear and their linearization can rarely serve for more or less consistent quantitative estimation (Sutulo & Guedes Soares 2011). At the same time, qualitative linear analysis may become useful as then the set (4) can be solved analytically and the solution still can capture the most important factors in a very transparent way.

Linearization can be performed in different ways resulting in very different linearized models. The general linearization problem was analyzed by Sutulo & Guedes Soares (2007). It is clear that in
the most general case i.e. linearizing over a biased domain or differentially around a biased equilibrium state the surge equation will remain coupled with the sway and yaw equations. However, this kind of linearization is typically required for the analysis of the local stability of motion (Sutulov 1985) not considered here. To analyze the equilibrium problem, it makes sense to linearize over a finite drift-and-rudder-angle domain symmetric with respect to the straight motion without wind. The resulting mathematical model will then be always inherently stable, the surge equation will be decoupled from the sway-yaw equations and can be analyzed separately. That analysis would result in an estimate of the equilibrium velocity of surge $u_0$ reachable at a given propeller rotation frequency. However, in the approximate linear analysis it is more reasonable to simply consider the equilibrium ship speed $V_0$ as an independent input parameter. Then, the linearized equilibrium sway-yaw equations take the form:

$$\frac{\rho}{2} V_0^2 \left(Y'_s + Y'_d \delta_R\right) + \frac{\rho A_L}{2} Y'_A(\beta_A) = 0,$$

$$\frac{\rho}{2} V_0^2 \left(N'_s + N'_d \delta_R\right) + \frac{\rho A_L}{2} L_{OA} N'_A(\beta_A) = 0,$$

(5)

where $\rho$, $\rho_A$ are the water and are densities respectively; $L$, $L_{OA}$ are the ship lengths: over the waterline and overall; $A_L$ is the lateral windage area; $T$ is the actual ship draught; $Y'_s$, $Y'_d$, $N'_s$, $N'_d$ are the so-called “hydrodynamic derivatives”; $Y'_A$, $N'_A$ are the dimensionless aerodynamic sway force and yaw moment.

Further, it can be assumed in the linear analysis that $\nu = -\beta$, $Y'_s = -Y'_d$, $N'_s = -N'_d$ and it is possible to neglect the difference between $L$ and $L_{OA}$. Finally, it makes sense to introduce abscissae of the points of application of the components of the sway force such as: $N'_A = x'_A Y'_A$, $N'_B = x'_B Y'_B$, $N'_D = x'_D Y'_D$.

Then, the equilibrium solution to the set (5) can be written as:

$$\beta_0 = -\kappa_\rho \kappa_A V_0^2 \frac{Y'_A(\beta_A)}{Y'_d} \frac{x'_A(\beta_A) - x'_d}{x'_B - x'_d},$$

$$\delta_{R0} = \kappa_\rho \kappa_A V_0^2 \frac{Y'_A(\beta_A)}{Y'_d} \frac{x'_A(\beta_A) - x'_d}{x'_B - x'_d},$$

(6)

where $\kappa_\rho = \rho_A / \rho$, $\kappa_A = A_L / (LT)$ and $V_A = V_d / V$, and the functions $Y'_A(\beta_A)$ and $x'_A(\beta_A)$ possess certain symmetry for most ships and it is then sufficient to define them for $\beta_A \in [0, \pi]$. The character of these functions depends considerably on the on the particular shape of the above-water part of the hull and should be preferably predicted with dedicated CFD computations or wind-tunnel tests. Less desirable but also possible is use of databases created after tests with characteristic models of various ship types (Isherwood 1972, Blendermann 1996). But for a qualitative analysis followed here even a simplest generic model for aerodynamic forces can be used. This model is defined by the following equations:

$$Y'_s = C_{Y_A} \sin \beta_A,$$

$$x'_A = x'_{40} + \frac{\xi - |\beta_A|}{2 \pi},$$

(7)

where $C_{Y_A}$ is a positive constant and $x'_{40}$ is the abscissa of the lateral windage centre.

The first of equations (7) looks in fact quite natural being the first term of the Fourier expansion for the periodic sway aerodynamic force. The second equation means that the sway aerodynamic force is applied at the windage area centre for the beam relative wind while the application point is shifted by a quarter of the ship length for a relative wind coming approximately from the bow or stern. This simple rule is surprisingly well confirmed by experimental data (Blendermann 1996) is shown in Figure 2 for the yaw moment coefficient as example.

It is seen directly from eqs. (6)–(7) that for all $\beta_A > 0$ (apparent wind from the portside) it will be $Y'_s > 0$ and, regarding that also $Y'_d > 0$, $x'_A - x'_d > 0$, $x'_B - x'_d > 0$, the resulting equilibrium parameters are: $\beta_0 < 0$ and $\delta_{R0} < 0$ i.e. the water drift is opposite to the air drift to create the countering hydrodynamic force and the rudder is deflected to reduce the air drift angle.

Figure 2. Aerodynamic yaw moment coefficient: generic model versus experimental data.
To get the problem formulation completely closed, it is necessary to link the air drift and the air speed of the vessel to the primary parameters i.e. to the relative absolute wind speed \( V_W = V_w / V \) and to either the heading angle \( \psi \) or to the course angle \( \chi \). As to the absolute wind angle, when the analysis covers all courses and headings with respect to the wind, it is possible to assume \( \chi_W = 0 \) without loss of generality. It means that the heading \( \psi = 0 \) will correspond to the stern wind. Then, the following relations can be easily established:

\[
\begin{align*}
\bar{V}_A \sin \beta_A &= \sin \beta - V_W \sin \psi, \\
\bar{V}_A \cos \beta_A &= \cos \beta - V_W \cos \psi.
\end{align*}
\]

The equations (8) are nonlinear and non-linearizable which makes the whole problem nonlinear even when it is based on the linearized equations (5). However, the problem can be greatly simplified if the earlier formulated controllability criterion based on the capability to sail arbitrary straight course is replaced by the requirement of the capability to sail at arbitrary air drift angle. Then, it will only be necessary to check for the value of the balancing rudder angle \( \delta_{R0} \) for all \( \beta_A \) from 0 to 180°. If for some value of the air drift angle \( \delta_{R0} > \delta_{adm} \) —the maximum admissible rudder angle, then the controllability is lost. It is typically assumed that \( \delta_{adm} = \delta_{max} - \Delta \delta \), where \( \delta_{max} > 0 \) is the maximum deflection angle and \( \Delta \delta > 0 \) is the deflection angle margin needed for the ship steering. In fact, introduction of a certain constant deflection margin is the simplest way to account for the sea waves excitation.

Equivalence of the primary and simplified approach seems to be clear intuitively but it can be even proved, at least for the most interesting case of a sufficiently strong wind when \( V_W > V \) that variation of the course angle from 0 to \( \pm \pi \) implies similar variations of the heading and course angles. Indeed, it is obvious that all three angles are zero in the case of stern wind and are all becoming equal to 180° for the head wind. All three angles pass through all intermediate values as long as they are linked by continuous functions. The continuity of this function defined by equations (6)–(8) can only be lost when the common denominator in (6) approaches zero. If it happens, the controllability will obviously be lost but in fact it is unlikely regarding the physical meaning of the involved dimensionless abscissae.

If the parameters \( \bar{V}_A \) and \( \beta_A \) are taken as primary, the equations (8) can be resolved with respect to the relative velocity \( V_W \) and the equilibrium ship heading \( \psi_0 \):

\[
V_W = \sqrt{1 - 2\bar{V}_A \cos (\beta_0 - \beta_A) + \bar{V}_A^2},
\]

\[
s_{\psi} = \frac{V}{V_W} \sin \beta_0 - \frac{V_A}{V_W} \sin \beta_A,
\]

\[
c_{\psi} = \frac{V}{V_W} \cos \beta_0 - \frac{V_A}{V_W} \cos \beta_A.
\]

(9)

where \( \beta_0 \) is defined by eq. (6).

In the particular case of zero water speed (station keeping) it will be \( \psi = \beta_A \) and the course angle is then not defined.

In the relatively light wind i.e. when \( V_W < V \) the ship cannot reach all values of the air drift angle but if it is forced into the equations (6), the resulting analysis can only bring conservative estimates.

Finally, it may happen that at a very strong wind the ship does not have sufficient thrust to sail against the wind and then the heading angle \( \psi = 0 \) will correspond to the course angle \( \chi = \pi \) or 180 degrees wind drift but this kind of controllability loss is not captured by the linear analysis.

2.4 Numerical results

In order to illustrate what kind of estimates can be obtained using the linearized model of steady motion in wind, calculations were performed for a hypothetical ship with \( \kappa_A = 5 \) which corresponds to sufficiently large relative windage area and with three values of the relative windage centre abscissa: \( x'_{d0} = 0, \pm 0.13 \). The necessary hydrodynamic derivatives defining the ship mathematical model are taken in the form:

\[
\begin{align*}
Y'_i &= Y'_{i0}(1 + b_i \tau'), \\
N'_i &= N'_{i0}(1 + b_j \tau'), \\
Y'_\delta &= -0.0586, \\
N'_\delta &= 0.0293,
\end{align*}
\]

(10)

where \( Y'_{i0} = -0.244, N'_{i0} = -0.0555 \) are the even keel values for Mariner ship, \( b_1 = 0.667 \), \( b_i = -0.27 Y'_i / N'_{i0} \) are the coefficients and \( \tau' \) is the ship trim as fraction of the draught, positive by the stern (Crane et al. 1989).

The meaning of introducing the variable trim is to vary the inherent directional stability of the ship: a trim by the stern increases the stability while a trim by the bow typically leads to directionally unstable ships.

Complete results are shown in Figures 3 and 4 for the even keel case and \( x'_{d0} = +0.13 \).

While qualitatively the main balancing parameters (the drift and rudder angles) depend on the air drift angle in the similar way for both weak and strong relative wind, quantitatively the results are very different. In the case of weak relative wind
Figure 3. Balancing kinematic parameters for weak relative wind: \( V_A = 15 \).

Figure 4. Balancing kinematic parameters for strong relative wind: \( V_A = 5 \).

i.e. when the air speed is only 50% higher than the ground speed the ship is balanced at any straight course at very small values of the rudder and drift angle. When the air speed is quintuple the ground speed the balancing drift angle is about 34 degrees and the required rudder angle can be as high as 35 degrees. If the latter is the maximum rudder angle for the given ship, it will indicate that, regarding that certain steering margin is necessary, the ship can lose controllability for \( \beta_A \) in the interval from approximately 110 to 150 degrees which corresponds to the stern quartering relative wind.

Bottom plots on both Figures show the balancing values of the course and heading angles. Obviously, the difference between these two angles is
insignificant in the case of weak wind and both dependencies are then substantially nonlinear. In strong wind the mentioned difference is pronounced but the \( \psi_0(\beta_A) \) curve looks closer to the straight line which means that the air drift angle is mainly determined by the heading of the ship with respect to wind.

Of course, the artificial assumption of keeping the constant relative wind speed or ship air speed \( \bar{V}_A \) implies variability of the absolute wind speed nondimensionalized by the ship speed \( \bar{V}_{W} \equiv V_{W}/V \) as shown by middle graphs in Figures 3 and 4. It must be noted that the character of dependencies of the absolute wind speed and of the steady course and heading values appeared to be very similar in all explored cases. That is why, only plots for the balancing values of the drift and rudder angle will be further presented for only \( \bar{V}_A = 5 \) and different position of the windage centre and other values of the imaginary ship trim. The even keel results for the middle and aft position of the windage centre are presented in Figure 5. It is seen that this parameter influences substantially the values of balancing parameters and, hence, the ship controllability in wind: when the windage area moves aft, the peak values of the balancing drift angle are somewhat decreasing but the required helms substantially increase reaching 60 and even 85 degrees. Of course, the linear mathematical model cannot be applied in such cases and the output data lose their quantitative sense but the indication to a possible loss of controllability is certain: aft position of the windage centre combined with aft relative wind makes the ship less controllable in strong wind. Also, it can be noticed that the maxima of the both balancing parameters are shifted to the halfwind regime.

Plots for the ship trimmed by the bow are shown in Figure 6 for all three positions of the windage centre. A ship with a bow trim typically demonstrates worse directional stability at somewhat better turning ability, and it is clear that in wind it is more prone to controllability loss than the same ship in even keel. At the same time, the most unfavourable interval of the air drift wind angles depends less on the configuration of the lateral windage area although the trend remains the same.

Results for a ship trimmed by the stern are presented in Figure 7. Here the effect of the balancing helm sign change slightly present in the even keel configuration becomes very pronounced but concerning the absolute values of the required rudder angle this case is roughly equivalent to the even keel condition.

Finally the case of the air speed inferior to the ground/water speed of the ship is illustrated in Figure 8. Of course, such case is useless from the practical viewpoint put it is of some interest because in this case the range of course/heading angles is limited by stern absolute winds while the relative wind may cover the full circle.

2.5 Possible generalizations and extensions

While the linear analysis demonstrated above turned out to be an efficient tool for revealing some qualitative properties and trends, in no way should it be used for more or less accurate quantitative estimates.

Possible ways of using more accurate and sophisticated mathematical models and algorithms are briefly outlined in this section.
Figure 6. Balancing kinematic parameters for strong relative wind $V_d = 5$, bow trim $\tau = -0.5$ and various position of the windage centre: top—$x_{\Delta 0} = +0.13$, middle—$x_{\Delta 0} = 0$, bottom—$x_{\Delta 0} = -0.13$.

Figure 7. Balancing kinematic parameters for strong relative wind $V_d = 5$, stern trim $\tau = 0.5$ and various position of the windage centre: top—$x_{\Delta 0} = +0.13$, middle—$x_{\Delta 0} = 0$, bottom—$x_{\Delta 0} = -0.13$.

Analytic solution of the type (6) is still possible when the ship mathematical model contains one non-linearity of the type $\beta/\beta$ in the sway force equation.

It must be noted that for both linear and simple “absolute square” models it is possible to estimate directly the critical value of the ship air speed provided the admissible value of the equilibrium rudder angle is defined. In the second case, the main equation with respect to the speed is of bi-quadratic type (Voytkunsky et al. 1973).
The mentioned simplified “quadratic” mathematical model was proposed by Pershitz (1983) and although it predicted well the turning ability of typical naval combatants, it could hardly serve for manoeuvring simulation of most vessels. More adequate and more complicated mathematical models do not permit analytical solutions and require more general numerical approach. It was found after some analysis that the most robust and straightforward way is direct simulation of the straight runs with varying courses using the differential equations (3) which, in addition, should be complemented with the equations for the steering gear and for the engine dynamics which can be described by the following general equations:

\[
\delta_R = F_R(\delta_R, \delta_R) \\
2\pi I_{pp}\dot{n} = Q_E(n, n^*) + Q_p(n, u),
\]

where \(\delta_R\) is the rudder order, \(F_R()\) is the nonlinear function accounting for the rudder angle and deflection rate saturations, \(I_{pp}\) is the moment of inertia of the system propeller–shaft–gear–engine, \(Q_E\) is the engine torque, \(n^*\) is the ordered propeller rotation frequency, and \(Q_p\) is the propeller torque.

But such simulation can only be performed if the system is closed by a feedback control law which can be taken in one of the following forms:

\[
\delta_R = k_p(\psi^* - \psi) - k_r r - k_{\psi}\Psi \\
\delta_R = k_x(\chi^* - \chi) - k_r r - k_{\chi}\chi,
\]

where \(\psi^*\) and \(\chi^*\) are the ordered heading and course angles respectively, \(k_p, k_r, k_{\psi}, k_{\chi}\) are the control gains, while \(\Psi\) and \(\chi\) are the auxiliary state variables defined by the equations:

\[
\dot{\Psi} = \psi^* - \psi; \\
\dot{\chi} = \chi^* - \chi.
\]

which means that these states represent time integrals of the heading/course angle errors, so both (13) and (14) represent the Proportional–Integral–Derivative (PID) control laws. Presence of the “integral” terms is highly desirable in this application as simpler PD control laws would produce static errors which may be quite considerable in presence of a constant wind disturbance.

While the heading PID control (13) is extremely common and implemented in many real-life controllers including practically all classic analogue autopilots, the direct course control, to our best

Figure 8. Balancing kinematic parameters for strong relative wind \(V_2 = 0.7\), even keel \(r = 0\) and \(\chi_{d0} = -0.13\).
knowledge, has not been applied so far. However, besides certain tradition this was caused by the simple fact that before introduction of GPS navigation the course angle (Course Over the Ground, COG) was practically unobservable while the heading could be easily picked up from the compass. Of course, in numerical simulation all state variables are observable and the course-angle steering is then preferable.

The control gains are typically chosen to optimize the control law in one way or another. In the course-keeping regime it is natural to minimize the heading error requiring, at the same time, restricted intensity and amplitude of the rudder deflections, so the final solution is obtained as some compromise.

However, regarding the special purpose of the simulations aimed at solving the equilibrium straight motion equations the quality of simulated steering is of minor importance and any gains values guaranteeing asymptotic stability of the closed-loop system can be applied. In that case, no sophisticated synthesis procedure is required and suitable gains can be found after several test simulations.

Obviously, a loop over all desired course angles from 0 to 180 degrees starting from the course corresponding to the pure bow or pure stern wind must be organized. A loss of controllability is captured when for some value of the desired course \( \chi \) the actual course angle \( \chi = \chi^* \) cannot be reached.

Presence of current may introduce additional complication into the analysis. If the current is steady and (quasi-)uniform, which is a good approximation in open sea, it does not affect the equilibrium heading but alters the course angle and the speed over the ground. Although the mathematical model only changes on the kinematical level, presence of two additional parameters (the current speed and direction) makes the analysis more complicated. In a somewhat different but similar problem of station keeping under combined action of wind and current (Sutulo 1995a) it turned out necessary to create Pareto sets to capture all worst combinations of the action of wind and current.

3 CONCLUSIONS

In the present study a general analysis of the implementation of environmental manoeuvring standards has been performed with main focus on the steady motion in wind. Possible formulations of this problem have been discussed and an analytic solution was presented in the case of the linearized manoeuvring model. The salient feature of the present analysis is characterised by explicit formulae permitting restoration of the ship heading and course angles as functions of the relative wind angle.

Computations carried out for a generic ship mathematical model demonstrated influence of such parameters as the relative wind speed, windage area configuration and directional stability of the ship in calm water.

Finally, approaches to future numerical analysis of the wind manoeuvrability of the ship on the basis of more complete and complex mathematical models were outlined.

Although results obtained in this study must not be used directly for establishing new manoeuvring criteria for ships in adverse weather conditions, they can serve as an appropriate guide and test bed in implementation of more detailed numerical schemes.

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REFERENCES


