Analysis of the numerical errors in the application of the 3D moving patch method to ship-to-ship interaction in shallow water

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ABSTRACT: The numerical errors of the moving patch variant of the 3D panel method applied to the problem of ship hydrodynamic interaction in shallow water are analysed in the case of the flat bottom. The patch moving method is compared to the mirror image method, which is considered as the reference method and results. However the mirror image method has the stringent limitation of not being applicable to the case of arbitrary bathymetry, which the moving patch method can handle. It is demonstrated that the errors of the moving patch algorithm remain insignificant when no dynamic re-panelling is applied. Tools and methods for reducing the numerical errors and smoothing the computed responses are investigated.

1 INTRODUCTION

Since the advent of the seminal work of Hess and Smith in the 1960’s (Hess & Smith, 1964), the constant panel method and its subsequent versions with extensions of functionality (Hess, 1990), on one hand, have been successfully applied to many areas of fluid dynamics (Newman, 1977; Kerwin, 1986; Kinnas, 1995; Sutulo & Guedes Soares, 2008), on the other hand, its drawback of insufficient accuracy in some cases has also been reported where higher order panel methods should be used (Hess, 1975).

However, despite its limited accuracy, the constant panel method, for its simplicity, robustness and relative ease of implementation, is still a valid and preferable tool particularly for the applications where the geometric configurations and the hydrodynamics is relatively less complex, provided that a sufficiently fine grid is used (Maniar, 1995; Katz & Plotkin, 2001) and a corresponding error analysis is carefully conducted (Ye & Fei, 2009).

A number of studies of the behaviour of the errors of the constant panel method for 2D problems can be found in the literature, and most of them focusing on the boundaries containing large curvatures as the linear convergence rate of the panel method is often observed in problems with smooth boundaries. The work of Kinnas & Hsin (1994) shows that the slow convergence of the low-order BEM on foils was due primarily to the effect of the local error at the sharp trailing edge. More recently a numerical study of an interior flow case was conducted by Ye & Fei (2009), who found that the convergence rate slows down greatly in problems with corners in the geometry.

In contrast with the 2D case, the error pattern of the method in the 3D case was studied and reported much less systematically. This is probably because it is considerably more difficult to mathematically model 3D problems and analytical solutions are often impossible.

This paper presents an error analysis for a 3D constant panel method applied to the calculation of the hydrodynamic interaction forces acting on ships in shallow water of constant depth. A simulation of an encounter scenario where both ships are moving on parallel courses is made. With the free surface effects neglected by using a double hull principle, the ship hulls are approximated with a set of flat quadrilateral elements remaining unchanged during the course of the simulation, while the bottom is modelled using the panelled moving patch method (Zhou et al., 2010) combined with dynamic meshing by means of the paving algorithm (Blacker & Stephenson, 1993). The results are compared with those produced by a similar algorithm but with the seabed modelled using the mirror image method (Sutulo & Guedes Soares, 2009), which was validated in Sutulo et al. (2012). The causes of the numerical errors are analysed and the influence of the local errors on the global solutions is presented. Finally, a number of solutions for reducing the errors are introduced, and their comparative advantages and disadvantages are discussed.

2 THEORETICAL FORMULATION AND PROBLEM DESCRIPTION

In the general problem of interacting surface ships moving in shallow water, the domain of perfect
Fluid is enclosed by the wetted ship surfaces, water surface, and the seabed. The total velocity potential is

$$\Phi = V_{\text{cur}} + \phi,$$  \hspace{1cm} (1)

where $V_{\text{cur}}$ is the velocity of a horizontal uniform onset flow, and $\phi$ is the perturbation potential which satisfies the Laplace equation

$$\Delta \phi = 0.$$  \hspace{1cm} (2)

With the low-Froude-number assumption, the free water surface effect is accounted for using a double hull model. Finally, the non-penetration boundary condition

$$\frac{\partial \phi}{\partial n} = V_r \cdot n$$  \hspace{1cm} (3)

is applied, where $n$ is the outward unity normal to the object in concern.

Based on the classic Hess and Smith method, two different models were implemented based on the classic Hess and Smith method. The mirror image technique for shallow water of constant depth is used in the first model, while the seabed is represented in the second model by distributing a layer of sources over a patch of finite size beneath the ship—the so-called panelled moving patch method. Therefore, the patches used on the seabed must be also included into the total surface $S$ in the following equation for the second implementation:

$$2\pi \sigma(M) + \int_S \sigma(P) \frac{\partial G(M,P)}{\partial n_M} dS(P) = f(M),$$  \hspace{1cm} (4)

where $M(x, y, z)$ is the observation field point, and $P$ is the source point belonging to the surface $S$.

After the equation (4) is solved for the source strength $\sigma$, the induced velocity $V_I$ and perturbation potential $\phi$ at the collocation point of a panel can be obtained by integrating the influences of all the source distributions. Then the pressure can be calculated by the unsteady Bernoulli equation

$$p = \rho \left[ -\frac{\partial \phi}{\partial t} + \frac{1}{2} (V_r^2 - V_p^2) \right],$$  \hspace{1cm} (5)

where

$$V_p = V_I - V_r$$  \hspace{1cm} (6)

in which the induced velocity $V_I$ and the relative local velocity $V_r$ are calculated using the following formulae:

$$V_I = \nabla \phi, V_r = V - V_{\text{cur}}$$  \hspace{1cm} (7)

where $V$ is the absolute local velocity of a point on the body surface. The total hydrodynamic inertial force $F_{pi}$ and moment $M_{pi}$ can then be easily calculated as:

$$F_{pi} = -\int_{S_i} \rho \text{n} \times dS, M_{pi} = -\int_{S_i} \rho \text{r} \times dS.$$  \hspace{1cm} (8)

Finally, subtracting the proper hydrodynamic inertial forces and moment from $F_{pi}$ and $M_{pi}$ gives the pure interaction loads on the ship: the surge force $X$, sway force $Y$, and yaw moment $N$ (Sutulo & Guedes Soares, 2008).

The moving patch method showed a very good agreement with the mirror image method in the case of one ship overtaking another on parallel courses in shallow water of constant depth, as long as the moving patch was meshed with square-shaped elements of uniform size (Zhou et al., 2010). However, such a meshing scheme is no longer the option for most of the interesting ship hydrodynamic interaction cases where more complex boundary configurations are involved, for instance, when the united patch is not rectilinear but arbitrary (Zhou et al., 2012).

Aiming to unleash the full potential of the panelled moving patch method, an automatic meshing algorithm based on the paving method by Blacker & Stephenson (1993) has been devised and integrated in the simulation system to dynamically discretize the bottom patches into quadrilateral elements. However, it has been observed that the results produced with the dynamically generated meshes contain significant numerical errors, and the time histories of the hydrodynamic interaction forces are often wrinkled in many cases.

3 ERROR ANALYSIS

The case of a ship overtaking another one on parallel courses in shallow water with the depth of 16 meters has been taken for the error analysis. Both Ship 1 (the overtaking ship) and Ship 2 (the overtaken ship) have the same ship form that has dimensions $L \times B \times T$ of 189.6 m $\times$ 31.6 m $\times$ 10.3 m, and the underwater hull surface has been pre-discretized into 542 flat quadrilateral panels. Ship 1 is moving at 4 m/s at the portside of Ship 2 moving at 1 m/s in the same direction, with a constant transverse distance of 36 meters between the two centre planes.

To model the shallow water, 7 reflections are used in the simulation with mirror image implementation. With the moving patch method, a patch with a size of $480 \times 200$ m is placed on the seabed.
right under each ship, with the patch and the ship being linked and moving together. These patches, or a patch union when they are intersected, are dynamically meshed into quadrilateral elements with a rough average side length of 8 meters.

Step-by-step error tracking leads to the calculation of the pressure on each panel using the unsteady Bernoulli equation, which contains two terms: the partial derivative of the velocity potential with respect to time $\frac{\partial \phi}{\partial t}$, and $(V^2 - V_{\infty}^2)$. Hence, errors in the velocity potential and in the induced velocity are the only causes of the pressure errors, and the algorithm of the Hess and Smith method is required to evaluate them. The time derivative of the velocity potential is zero for the steady flow while the second term is always present. In the present study, the flow is always assumed to be quasi-steady, i.e. only the quadratic term of the Bernoulli equation is taken into account.

3.1 Induced velocities

As the focus of this study is the convergence of the panel method instead of the actual force information, there is no need to analyse the numerical results for both ships, and only those for Ship 1 (the overtaking ship) are presented here.

Figure 1 shows the absolute difference in the results by two methods for the x-, y-, z-components of the induced velocity $V_x, V_y, V_z$. It is clear that there are numerical errors in every component of the induced velocity.

In order to reflect the magnitude of the differences, the results of the mirror image method are used as benchmark and the errors in the results produced by the panelled moving patch are compared to the current velocity, as:

$$e_{VI} = \frac{(V_I)_{pmp} - (V_I)_{mi}}{V_{cur}} \times 100\%,$$

where $e_{VI}$ is the error relative to the current velocity, $(V_I)_{pmp}$ the results for the induced velocity produced by the panelled moving patch method, and $(V_I)_{mi}$ that by the mirror image method. This way of processing is reasonable and meaningful because according to the steady flow Bernoulli equation the pressure on a panel is determined by the local fluid velocity and the velocity of the fluid afar off. The processed results are shown in Figure 2. It can be seen that the errors vary within the range of $-1\%$ to $0.5\%$ of the current velocity.

3.2 Induced potential

As indicated in the unsteady Bernoulli equation, calculating the contribution of unsteadiness to the pressure requires evaluation of the time derivative of the velocity potential. To do such, the potential felt at a panel at several previous time instants, one at least, i.e. the first order upwind scheme is employed, must be used. The differences between the results by the two methods for the velocity potential at panels are plotted in Figure 3 for the first four instants (computation steps).

Despite the obvious discrepancy between results by difference methods, the numerical errors seem to be consistent in magnitude over time, see Figure 3, and they may result in satisfactory results for the time derivative of the velocity potential. But sadly,
as shown in Figure 4, the velocity derivatives calculated from such results are very different from one instant to the next.

3.3 Forces on panels

Examining the errors in the numerical results for the induced velocity and potential still do not explain the numerical jumps in the time histories of the hydrodynamic interaction forces acting on the ship. There are two reasons for that: 1) the force acting on a panel depends not only on the pressure but also on its area, and—for the yaw moment—on its location, and 2) the pressure acting on portside is partially cancelling that on the starboard. As a result, the relative error for the force may be larger.

To reveal the influence of each kind of error on the final results, the same overtaking scenario is simulated using each method in both quasi-steady and unsteady flow mode. While in the first case the resulting error is only caused by the error in the induced velocity, in the second case the error in the potential also contributes.

As shown in Figures 5 and 6, the errors in the results obtained in the unsteady mode for the panel sway forces are significantly greater than those obtained in the steady mode, which suggests that

Figure 3. Difference between the results by the two methods for the velocity potential at body panels at the first four instants.

Figure 4. Time derivative of the velocity potential at each body panel at the 2nd, 3rd, 4th instants respectively.

Figure 5. Panel sway force, quasi-steady flow mode.

Figure 6. Panel sway force, unsteady flow mode.
the numerical errors associated with the first term of the Bernoulli equation may have more influence on the global solution. Similar results for the panel yaw moments are obtained and presented in Figures 7 and 8.

3.4 Numerical errors caused by geometric characteristics of grid elements

The promising agreement obtained between the mirror image method and the panelled moving patch method with all square-shaped elements on the seabed (Zhou et al., 2010) implies that the errors might have resulted from the geometrical characteristics of the grids automatically generated by the meshing algorithm during the simulation. This was verified by a simulation of the same scenario but with an extended transverse distance between two ships and with an asymmetric seabed patching scheme as shown in Figure 9.

The results calculated with such a configuration are shown in Figures 10 and 11, where the hydrodynamic interaction forces are nondimensionalised by:

\[
\xi' = \frac{\xi_1 - \xi_2}{L_1 + L_2},
\]

(10)
This observation indicates that, as long as the meshes generated at consecutive computation steps share a geometrical similarity, smooth curves of force histories can be obtained. This is because the value of any intermediate quantity for a panel, e.g. the induced velocity or the pressure, evaluated at the control point (centroid) of the panel is taken as the value for any point elsewhere on the panel; such a uniform representation of quantities bears an error with the actual distributions in a pattern specific to the geometric characteristics of the panel for a given flow state, and this error pattern remains the same or similar at the next time instant if the geometric characteristics do not change and provided the change of flow properties between the two instants is relatively small. As a result, the quantities at two consecutive steps would have errors of the same order, and in turn a smooth curve of time history is produced.

Unfortunately, in general it is unlikely that the two sets of grid elements generated at different moments can share a geometrical similarity because the border of the patch union keeps changing as the overtaking process goes.

4 ERROR REDUCTION PROCEDURES

The above analysis shows that more significant numerical errors are associated with the first of the two terms of the Bernoulli equation; therefore it is natural to employ finite differencing schemes to reduce the amplitude of these errors. Since the upwind schemes are preferable for real-time simulations compared to central schemes as only the results of previous steps are needed, the 3rd order and the 5th order upwind schemes proposed by Fornberg (1988) were used. The derivative of an arbitrary function \( f(p) \) is the approximated as:

\[
\frac{d}{dp} = \frac{f(p+h) - f(p-h)}{2h},
\]

where \( h \) is the mesh size.

This is the 3rd order scheme.

\[
\frac{d}{dp} = \frac{5f(p+h) - 2f(p) - 3f(p-h)}{2h},
\]

where \( h \) is the mesh size.

This is the 5th order scheme.

Numerical results produced with high order upwind schemes for the hydrodynamic interaction forces acting on Ship 1 in the same overtaking scenario as before are plotted in Figures 12, 13, and 14. The results by the mirror image...
method and the panelled moving patch with the first order upwind scheme are also presented as reference. It is clearly shown that including results of more steps does not offer a more accurate finite difference scheme for the approximation of time derivative, on the contrary, the higher is the order of the upwind scheme, the more amplified are the numerical errors.

With the hope that the numerical errors in the results of the preceding instants might somehow cancel that of the succeeding ones, the 4th and the 8th order central schemes (Fornberg, 1988) have also been implemented:

\[
\begin{align*}
 f'(p) & = \frac{1}{h_p} \left[ \frac{1}{12} f(p_{-2}) - \frac{2}{3} f(p_{-1}) + \frac{2}{3} f(p_1) - \frac{1}{12} f(p_2) \right] + O(h_p^4) \\
 f'(p) & = \frac{1}{h_p} \left[ \frac{1}{280} f(p_{-4}) - \frac{4}{105} f(p_{-3}) + \frac{1}{5} f(p_{-2}) - \frac{4}{5} f(p_{-1}) + \frac{1}{5} f(p_2) + \frac{4}{105} f(p_3) - \frac{1}{280} f(p_4) \right] + O(h_p^5).
\end{align*}
\]

Moreover, similarly to the upwind schemes, central schemes with higher orders worsen the results: see Figures 15, 16, and 17. Higher order upwind and central schemes usually produce approximations of derivatives with a better accuracy; however, they are of no help in this case as the values for the induced potentials contain inaccuracies themselves.

Since the inspection revealed that the results were deteriorated primarily due to the weights in the high order differencing scheme models, a 6-point unweighted moving average model (Velleman et al., 1981), also known as the simple moving average,

\[
f(p) = \frac{1}{6} \left[ f(p_{-3}) + f(p_{-2}) + f(p_{-1}) + f(p_1) + f(p_2) + f(p_3) \right]
\]

has been used as an attempt to smoothen the numerical results and indeed in this case, the numerical jumps in the time histories of the forces are nicely removed: see Figures 18, 19, and 20.
Figure 15. Central scheme, surge force on Ship 1.

Figure 16. Central scheme, sway force on Ship 1.

Figure 17. Central scheme, yaw moment on Ship 1.

Figure 18. 6-point moving average, surge force on Ship 1.

Figure 19. 6-point moving average, sway force on Ship 1.

Figure 20. 6-point moving average, yaw moment on Ship 1.
The simple moving average, as a makeshift solution, results in an outcome for this application better than higher order differencing schemes. This again confirms that high order differencing schemes help to improve the precision of the results only under the condition that the accuracy of the scheme is guaranteed.

However, moving average is in no way an ideal solution to the numerical problem not only because it may reduce the amplitude to some extent in some cases: (1) if, as conventionally in science and engineering, an equal number of nodes on either side of the central value are taken for calculating the moving average, the hydrodynamic interaction simulation can be no longer real-time as computation for the present instant requires future information; (2) or, if alternatively the simple moving average is calculated as the mean of the previous n data, as is typical in finance, a time shift will be introduced.

5 CONCLUSIONS

An error analysis of the classic Hess and Smith method was conducted for a case of real-time simulation of ship hydrodynamic interaction forces in shallow water of constant depth where dynamic meshing of the fluid boundary is required. Based on a systematic comparison of numerical results produced with and without dynamic meshing, the following conclusions can be drawn:

1. The numerical results produced with dynamically generated meshes may contain significant numerical errors. These errors originate from the lack of geometrical similarity between the grid elements generated at consecutive time steps.
2. The errors contained in the numerical values of the potential complicate obtaining accurate derivatives of the potential. Unfortunately, high order finite differencing schemes are of no help in the case of dynamic meshing.
3. The limited accuracy of the values for the induced velocities also contributes to the errors in the global solution.
4. The simple moving average method can effectively improve the global solution, but is not applicable to real-time simulation.
5. For the problems requiring dynamic meshing, a new formulation of the panel method is to be devised in which the source distributions are more accurate but not so complex as to cause difficulty in generating the meshing dynamically.

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